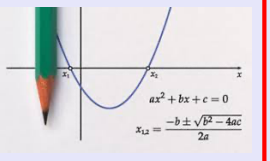
Math 125

Spring 2022

Lecture 25



Class QZ 18

Solve by square-root method.

$$(2x-3)^2+10=-15$$

$$(2x-3)^2=-15-10$$

$$(2x-3)^2=-25$$

$$2x-3=\pm\sqrt{-25}$$

$$2x=3\pm5i$$

$$x=\frac{3}{2}\pm\frac{5}{2}i$$

$$\left\{\frac{3}{2}\pm\frac{5}{2}i\right\}$$

Solve $(5x-3)(2x+5)=63$ by using the quadratic formula.

we need to have $ax^2+bx+c=0$, $a\neq 0$.

↳ Soil & simplify, and write in $ax^2+bx+c=0$ form.

$$10x^2+25x-6x-15-63=0$$

$$10x^2+19x-78=0$$

$$b^2-4ac=19^2-4(10)(-78)=\boxed{3481}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-19\pm\sqrt{3481}}{2(10)}=\frac{-19\pm 59}{20}$$

$$x=\frac{-19+59}{20}=\frac{40}{20}=2$$

$$x=\frac{-19-59}{20}=\frac{-78}{20}=\frac{-39}{10}$$

$$\Rightarrow \left\{\frac{-39}{10}, 2\right\}$$

Determine the type of solutions for

$$25x^2-30x+11=0$$

↳ we need value of the discriminant.

$$b^2-4ac=(-30)^2-4(25)(11)$$

$$=900-1100=-200$$

When $b^2-4ac<0 \Rightarrow$ Two complex number solutions.

Determine the type of Solutions for

$9x^2 + 16 = 24x$

↳ we need $b^2 - 4ac$.

$ax^2 + bx + c = 0$

$9x^2 + 16 - 24x = 0$

order

$9x^2 - 24x + 16 = 0$

$a = 9, \quad b = -24, \quad c = 16$

$b^2 - 4ac = (-24)^2 - 4(9)(16) = 0$

when $b^2 - 4ac = 0 \Rightarrow$ we get one repeated real number Solution.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-24) \pm \sqrt{0}}{2(9)} = \frac{24 \pm 0}{18} = \frac{24}{18} = \boxed{\frac{4}{3}}$ only one Solution

Find a quadratic equation in $ax^2 + bx + c = 0$ Form with Solutions -1 and $\frac{3}{5}$.

Solns: $x = -1$ $x = \frac{3}{5}$

Clear Fractions, Make RHS = 0 $x + 1 = 0$ $5x = 3$, $5x - 3 = 0$

Factors $(x + 1)(5x - 3) = 0$

Foil & Simplify $5x^2 - 3x + 5x - 3 = 0$

$\boxed{5x^2 + 2x - 3 = 0}$

Find a quadratic equation in $ax^2+bx+c=0$ Form with Solutions $-5 \pm 3\sqrt{2}$.

Solutions $x=-5+3\sqrt{2}$ $x=-5-3\sqrt{2}$

Make RHS=0 $x+5-3\sqrt{2}=0$ $x+5+3\sqrt{2}=0$

Factors $(x+5-3\sqrt{2})(x+5+3\sqrt{2})=0$

Conjugates

$(x+5)^2-(3\sqrt{2})^2=0$

$(x+5)(x+5)-9\cdot 2=0$

Foil & Simplify

✓ $\boxed{x^2+10x+7=0}$

Find a quadratic equation in $ax^2+bx+c=0$ Form with Solutions $\frac{3}{5} \pm \frac{4}{5}i$.

Solutions $x=\frac{3}{5}+\frac{4}{5}i$ $x=\frac{3}{5}-\frac{4}{5}i$

Clear Fractions $5x=3+4i$ $5x=3-4i$

Make RHS=0 $5x-3-4i=0$ $5x-3+4i=0$

Factors $(5x-3-4i)(5x-3+4i)=0$

Conjugates

$(5x-3)^2-(4i)^2=0$

$(5x-3)(5x-3)-16i^2=0$

Foil, then Simplify

$\boxed{25x^2-30x+25=0}$

Divide by 5

✓✓ $\boxed{5x^2-6x+5=0}$

Make a perfect-square

$x^2 - 18x + (9)^2 = (x-9)^2$

$\frac{1}{2} \cdot 18 = 9$

$x^2 - 18x + 81 = (x-9)^2$

Make a Perfect-Square:

$x^2 + \frac{4}{5}x + \left(\frac{2}{5}\right)^2 = \left(x + \frac{2}{5}\right)^2$

$\frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$

$x^2 + \frac{4}{5}x + \frac{4}{25} = \left(x + \frac{2}{5}\right)^2$

Solve by Completing the square method:

$x^2 + 10x + 24 = 0$

$x^2 + 10x + 5^2 = -24 + 5^2$

$\frac{1}{2} \cdot 10 = 5$

$(x+5)^2 = 1$

SRM

$x + 5 = \pm\sqrt{1}$

$x + 5 = \pm 1$

$x = -5 \pm 1$

$x = -5 + 1 = -4$

$x = -5 - 1 = -6$

$\{-6, -4\}$

Solve by Completing the Square method:

$$x^2 - 8x + 20 = 0$$

$$x^2 - 8x + 4^2 = -20 + 4^2$$

$\frac{1}{2} \cdot 8 = 4$

$$(x-4)^2 = -4$$

Now S.R.M.

$$x - 4 = \pm\sqrt{-4}$$

$$x = 4 \pm 2i$$

$$\{4 \pm 2i\}$$

Solve by Completing the Square method:

$$x^2 - 3x - 10 = 0$$

$$10 + \frac{9}{4} = \frac{40}{4} + \frac{9}{4}$$

$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 10 + \left(\frac{3}{2}\right)^2$$

$\frac{1}{2} \cdot 3 = \frac{3}{2}$

$$\left(x - \frac{3}{2}\right)^2 = \frac{49}{4}$$

by S.R.M.

$$x - \frac{3}{2} = \pm\sqrt{\frac{49}{4}}$$

$$x = \frac{3}{2} \pm \frac{7}{2}$$

$$x = \frac{3}{2} + \frac{7}{2} = \frac{10}{2} = 5$$

$$x = \frac{3}{2} - \frac{7}{2} = \frac{-4}{2} = -2$$

$$\{-2, 5\}$$

Solve by Completing the Square method:

$$2x^2 - 3x - 5 = 0$$

Hint: Lead. Coef. must be 1.

Divide by 2

$$x^2 - \frac{3}{2}x - \frac{5}{2} = 0$$

$\frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

$$x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{5}{2} + \left(\frac{3}{4}\right)^2$$

$$\frac{5 \cdot 8}{2 \cdot 8} + \frac{9}{16} = \frac{40}{16} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{49}{16}$$

by S.R.M.

$$x - \frac{3}{4} = \pm\sqrt{\frac{49}{16}}$$

$$x = \frac{3}{4} \pm \frac{7}{4}$$

$$x = \frac{3+7}{4} = \frac{10}{4} = \boxed{\frac{5}{2}}$$

$$x = \frac{3-7}{4} = \frac{-4}{4} = \boxed{-1}$$

$$\left\{-1, \frac{5}{2}\right\}$$

Solving equations in Quadratic Form:

Solve $(\sqrt{x} - 1)^2 - 8(\sqrt{x} - 1) + 15 = 0$

Let $u = \sqrt{x} - 1$ $\Rightarrow$ $u^2 - 8u + 15 = 0$

$$(u - 5)(u - 3) = 0$$

$u = 5$ $\quad$ $u = 3$

For $u = 5$

$\sqrt{x} - 1 = 5$ $\quad$ $\sqrt{x} = 6$ $\quad$ $(\sqrt{x})^2 = 6^2$ $\quad$ $\boxed{x = 36}$

For $u = 3$

$\sqrt{x} - 1 = 3$ $\quad$ $\sqrt{x} = 4$ $\quad$ $(\sqrt{x})^2 = 4^2$ $\quad$ $\boxed{x = 16}$

$$\{16, 36\}$$

Solve $x^4 - 5x^2 - 36 = 0$

$(x^2)^2 - 5x^2 - 36 = 0$

Let $u = x^2$

$u^2 - 5u - 36 = 0$

$(u-9)(u+4) = 0$

$u = 9$ $\quad u = -4$

$\{\pm 2i, \pm 3\}$

If $u = 9$

$x^2 = 9 \rightarrow x = \pm\sqrt{9} \rightarrow \boxed{x = \pm 3}$

If $u = -4$

$x^2 = -4 \rightarrow x = \pm\sqrt{4} \rightarrow \boxed{x = \pm 2i}$

Your turn: $x^6 - 28x^3 + 27 = 0$

Hint:

Let $u = x^3$

$u^2 = (x^3)^2 = x^6$

$u^2 - 28u + 27 = 0$

$(u-1)(u-27) = 0$

$u = 1$ $\quad u = 27$

when $u = 1$

$x^3 = 1$

$x = \sqrt[3]{1}$

$\boxed{x = 1}$

when $u = 27$

$x^3 = 27$

$x = \sqrt[3]{27}$

$\boxed{x = 3}$

$\{1, 3\}$

Your turn:

Solve $x^{\frac{2}{3}} - 6x^{\frac{1}{3}} + 8 = 0$

$\rightarrow u^2 - 6u + 8 = 0$

Hint:

Let $u = x^{\frac{1}{3}}$

$u^2 = \left[x^{\frac{1}{3}}\right]^2 = x^{\frac{1}{3}\cdot 2} = x^{\frac{2}{3}}$

$(u-2)(u-4) = 0$

$u = 2 \quad u = 4$

when $u = 2$

$x^{\frac{1}{3}} = 2$

$\sqrt[3]{x} = 2$

$(\sqrt[3]{x})^3 = (2)^3$

$\boxed{x = 8}$

when $u = 4$

$x^{\frac{1}{3}} = 4$

$\sqrt[3]{x} = 4$

$(\sqrt[3]{x})^3 = (4)^3$

$\boxed{x = 64}$

$\{8, 64\}$

Solve: $x^{\frac{2}{5}} + 2x^{\frac{1}{5}} + 1 = 0$

Let $u = x^{\frac{1}{5}}$

$u^2 = x^{\frac{2}{5}}$

$\rightarrow u^2 + 2u + 1 = 0$

$(u+1)(u+1) = 0$

$u = -1$

when $u = -1$

$x^{\frac{1}{5}} = -1$

$\sqrt[5]{x} = -1$

$(\sqrt[5]{x})^5 = (-1)^5$

$\boxed{x = -1}$

$\{-1\}$

Solve $3\left(\frac{x-1}{x+1}\right)^2 - 2\left(\frac{x-1}{x+1}\right) - 5 = 0$

Let $u = \frac{x-1}{x+1}$

$3u^2 - 2u - 5 = 0$

$a \quad b \quad c$

$b^2 - 4ac = (-2)^2 - 4(3)(-5) = 64$

$u = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{64}}{2(3)}$

$= \frac{2 \pm 8}{6}$

$u = \frac{2+8}{6} = \frac{10}{6} = \frac{5}{3}$

$u = \frac{2-8}{6} = \frac{-6}{6} = -1$

when $u = \frac{5}{3}$

$\frac{x-1}{x+1} = \frac{5}{3}$

$5(x+1) = 3(x-1)$

$5x+5 = 3x-3$

$5x-3x = -3-5$

$2x = -8$ $\quad x = -4$

when $u = -1$

$\frac{x-1}{x+1} = -1$ $\quad \frac{x-1}{x+1} = \frac{-1}{1}$

$x-1 = -1(x+1)$

$x-1 = -x-1$

$x+x = -1+1$

$2x = 0$ $\quad x = 0$

$\{0, -4\}$

Solve $\frac{1}{x^2} + \frac{8}{x} + 15 = 0$

$\left(\frac{1}{x}\right)^2 + 8\left(\frac{1}{x}\right) + 15 = 0$

Let $u = \frac{1}{x}$

$u^2 + 8u + 15 = 0$

$u^2 + 8u + 4^2 = -15 + 4^2$

$(u+4)^2 = 1$

by S.R.M.

$u+4 = \pm\sqrt{1}$

$u = -4 \pm 1$

$u = -4+1 = -3$

$u = -4-1 = -5$

when $u = -3$

$\frac{1}{x} = -3 \rightarrow x = \frac{-1}{3}$

when $u = -5$

$\frac{1}{x} = -5 \rightarrow x = \frac{-1}{5}$

$\left\{\frac{-1}{5}, \frac{-1}{3}\right\}$

Class QZ 19

Solve $2x(x-2)=1$ by the quadratic formula.

$$2x^2-4x-1=0$$

$a=2$, $b=-4$, $c=-1$

$\Rightarrow b^2-4ac=(-4)^2-4(2)(-1)=16+8=\boxed{24}$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-4)\pm\sqrt{24}}{2(2)}=\frac{4\pm\sqrt{4}\sqrt{6}}{4}$$

$$=\frac{4\pm2\sqrt{6}}{4}=\frac{2(2\pm\sqrt{6})}{4}=\frac{2\pm\sqrt{6}}{2}$$

$$\left\{\frac{2\pm\sqrt{6}}{2}\right\}$$